

# PROBLEM OF THE MONTH #2

FEBRUARY 2023

**Directions:** Write a complete solution to the problem below showing all work. Your paper must have your name, W#, and Southeastern email address. Solutions are to be sent as a SINGLE PDF FILE to the submission address [talwissubmissions@selu.edu](mailto:talwissubmissions@selu.edu), with the subject heading of the email as Problem of the Month #2, February 2023, by 11:59 p.m., **Tuesday, February 28**. No late papers will be accepted.

All papers with a correct solution will be entered in a drawing for a great prize! Anyone can submit solutions, but only currently enrolled students are eligible for prizes.

Questions concerning the problem of the month should be sent to either Dr. Tilak de Alwis ([tdealwis@selu.edu](mailto:tdealwis@selu.edu)), or Dr. Dennis Merino ([dmerino@selu.edu](mailto:dmerino@selu.edu))

## **PROBLEM: *Elementary Number Theory***

(a) Suppose  $x, y, z$  are three positive integers such that  $3x = 4y = 5z$ . Find the least possible value of  $x^2 + y^2 + z^2$ . Justify your answer.

(b) Suppose  $a_1, a_2, \dots, a_n$  are  $n$  positive integers such that  $a_1x_1 = a_2x_2 = \dots = a_nx_n$  where  $n$  is some natural number greater than 1. What is the least possible value of  $x_1^2 + x_2^2 + \dots + x_n^2$ ? Justify your answer.

*Note:* Partial answers might still be considered. So all submissions are welcome!