

PROBLEM OF THE MONTH #2

SEPTEMBER 2023

Directions: Write a complete solution to the problem below showing all work. Your paper must have your name, W#, and Southeastern email address. Solutions are to be sent as a SINGLE PDF FILE to the submission address talwissubmissions@selu.edu, with the subject heading of the email as Problem of the Month #2, September 2023, by 11:59 p.m., **Friday, September 29**. No late papers will be accepted.

All papers with a correct solution will be entered in a drawing for a great prize! Anyone can submit solutions, but only currently enrolled students are eligible for prizes.

Questions concerning the problem of the month should be sent to either Dr. Tilak de Alwis (tdealwis@selu.edu), or Dr. Dennis Merino (dmerino@selu.edu)

PROBLEM: *It's all about limits!*

- (a) Without using any derivative techniques, find the exact and simplified value of the following limit:

$$\lim_{x \rightarrow \pi/2} \frac{3\sin(8x) + x\cos(11x) + 2\cos(11x)}{3\sin(8x) - x\cos(11x) - 2\cos(11x)}$$

- (b) Use any tool to draw a large and clear graph of the following function, for $-\pi < x < \pi$:

$$f(x) = \ln(1 - \cos x) - \frac{1}{2} \ln(\sin^2 x) (1 + \cos x)$$

- (c) Is f continuous at $x = 0$? Give the precise mathematical reason, using limit calculations. If discontinuous at $x = 0$, name the type of discontinuity. For this part (c), you may use any method of calculus.